No–scale supergravity and the Multiple Point Principle

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Abstract

We review symmetries protecting a zero value for the cosmological constant in no–scale supergravity and reveal the connection between the Multiple Point Principle, no–scale and superstring inspired models.

1. Introduction

Nowadays the existence of a tiny energy density spread all over the Universe (the cosmological constant), which is responsible for its accelerated expansion, provides the most challenging problem for modern particle physics. A fit to the recent data shows that $\Lambda \sim 10^{-123} M_{Pl}^4 \sim 10^{-55} M_Z^4$ [1]. At the same time the presence of a gluon condensate in the vacuum is expected to contribute an energy density of order $\Lambda_{QCD}^4 \simeq 10^{-74} M_{Pl}^4$. On the other hand if we believe in the Standard Model (SM) then a much larger contribution $\sim v^4 \simeq 10^{-62} M_{Pl}^4$ must come from the electroweak symmetry breaking. The contribution of zero–modes is expected to push the vacuum energy density even higher up to $\sim M_{Pl}^4$. Thus, in order to reproduce the observed value of the cosmological constant, an enormous cancellation between the various contributions is required. Therefore the smallness of the cosmological constant should be considered as a fine-tuning problem. For its solution new theoretical ideas must be employed.

Unfortunately the cosmological constant problem can not be resolved in any available generalization of the SM. An exact global supersymmetry (SUSY) ensures zero value for the vacuum energy density. But in the exact SUSY limit bosons and fermions from one chiral multiplet get the

same mass. Soft supersymmetry breaking, which guarantees the absence of superpartners of observable fermions in the 100 GeV range, does not protect the cosmological constant from an electroweak scale mass and the fine-tuning problem is re-introduced.

It was argued many years ago that soft breaking of global supersymmetry at low energies could be consistent with a zero value for the cosmological constant in the framework of $N = 1$ supergravity (SUGRA) models [2]. Moreover there is a class of models (so called no–scale supergravity) where the vacuum energy density vanishes automatically [3]. It happens because no–scale models possess an enlarged global symmetry. Even after breaking, this symmetry still protects zero vacuum energy density at the tree level. All vacua in the no–scale models are degenerate, which provides a link between no–scale supergravity and the Multiple Point Principle (MPP) [4]. MPP postulates that in Nature as many phases as possible, which are allowed by the underlying theory, should coexist. On the phase diagram of the theory it corresponds to the special point – the multiple point – where many phases meet. According to the MPP, the vacuum energy densities of these different phases are degenerate at the multiple point.

This article is organized as follows: in section 2 we describe the structure of ($N = 1$) SUGRA models; in section 3 we study symmetries protecting the zero value of the cosmological constant in the no-scale models ignoring the superpotential; the no–scale models with a non–trivial superpotential are considered in section 4. The connection between the MPP, no–scale and superstring inspired models is discussed in section 5.

2. $N = 1$ supergravity

The full $N = 1$ SUGRA Lagrangian [3],[5] is specified in terms of an analytic gauge kinetic function $f_a(\phi_M)$ and a real gauge-invariant Kähler function $G(\phi_M, \bar{\phi}_M)$, which depend on the chiral superfields ϕ_M. The function $f_a(\phi_M)$ determines the kinetic terms for the fields in the vector supermultiplets and the gauge coupling constants $Ref_a(\phi_M) = 1/g_a^2$, where the index a designates different gauge groups. The Kähler function is a combination of two functions

$$G(\phi_M, \bar{\phi}_M) = K(\phi_M, \bar{\phi}_M) + \ln |W(\phi_M)|^2 \,, \tag{1}$$

where $K(\phi_M, \bar{\phi}_M)$ is the Kähler potential whose second derivatives define the kinetic terms for the fields in the chiral supermultiplets. $W(\phi_M)$ is the complete analytic superpotential of the considered SUSY model. In this article standard supergravity mass units are used: $\dfrac{M_{Pl}}{\sqrt{8\pi}} = 1$.

The SUGRA scalar potential can be presented as a sum of $F-$ and D–terms $V = V_F + V_D$, where the F–part is given by [3],[5]

$$
\begin{gathered}
V_F(\phi_M, \bar{\phi}_M) = e^G \left(\sum_{M,\, \bar{N}} G_M G^{M\bar{N}} G_{\bar{N}} - 3 \right) , \\
G_M \equiv \partial_M G \equiv \partial G / \partial \phi_M , \qquad G_{\bar{M}} \equiv \partial_{\bar{M}} G \equiv \partial G / \partial \phi^*_M , \\
G_{\bar{N}M} \equiv \partial_{\bar{N}} \partial_M G = \partial_{\bar{N}} \partial_M K \equiv K_{\bar{N}M} .
\end{gathered}
\tag{2}
$$

The matrix $G^{M\bar{N}}$ is the inverse of the Kähler metric $K_{\bar{N}M}$. In order to break supersymmetry in $(N = 1)$ SUGRA models, a hidden sector is introduced. It contains superfields (h_m), which are singlets under the SM $SU(3) \times SU(2) \times U(1)$ gauge group. If, at the minimum of the scalar potential (2), hidden sector fields acquire vacuum expectation values so that at least one of their auxiliary fields

$$
F^M = e^{G/2} \sum_{\bar{P}} G^{M\bar{P}} G_{\bar{P}} \tag{3}
$$

is non-vanishing, then local SUSY is spontaneously broken. At the same time a massless fermion with spin 1/2 – the goldstino – is swallowed by the gravitino which becomes massive $m_{3/2} =< e^{G/2} >$. This phenomenon is called the super-Higgs effect.

It is assumed that the superfields of the hidden sector interact with the observable ones only by means of gravity. Therefore they are decoupled from the low energy theory; the only signal they produce is a set of terms that break the global supersymmetry of the low-energy effective Lagrangian of the observable sector in a soft way. The size of all soft SUSY breaking terms is characterized by the gravitino mass scale $m_{3/2}$.

In principle the cosmological constant in SUGRA models tends to be huge and negative. To show this, let us suppose that, the Kähler function has a stationary point, where all derivatives $G_M = 0$. Then it is easy to check that this point is also an extremum of the SUGRA scalar potential. In the vicinity of this point local supersymmetry remains intact while the energy density is $-3 < e^G >$, which implies the vacuum energy density must be less than or equal to this value. In general enormous fine-tuning is required to keep the cosmological constant around its observed value in supergravity theories.

3. $SU(1,1)$ and $SU(n,1)$ symmetries in the no–scale models

We know that the smallness of the parameters in a physical theory can usually be related to an almost exact symmetry. Since the cosmological constant is extremely tiny, one naturally looks for a symmetry reason to guarantee its smallness in supergravity. In the simple case when there is only one singlet chiral multiplet $\hat{z}$, the scalar potential can be written as

$$V(z,\bar{z}) = 9e^{4G/3}G_{z\bar{z}}\left(\partial_z\partial_{\bar{z}}e^{-G/3}\right) . \tag{4}$$

In order that the vacuum energy density of $V(z,\bar{z})$ should vanish, we must either choose some parameters inside G to be fine–tuned or, alternatively, demand that the Kähler function G satisfies the differential equation $\partial_z\partial_{\bar{z}}e^{-G/3}=0$, whose solution is [6]:

$$G = -3\ln\left(f(z)+f^*(\bar{z})\right) . \tag{5}$$

For the Kähler function given by Eq. (5), fine–tuning is no longer needed for the vanishing of the vacuum energy, since the scalar potential is flat and vanishes at any point z. The kinetic term for the field z is then given by

$$L_{kin} = \frac{3\partial_z f(z)\partial_{\bar{z}} f^*(\bar{z})}{(f(z)+f^*(\bar{z}))^2}\left|\partial_\mu z\right|^2 = \frac{3\left|\partial_\mu f(z)\right|^2}{(f(z)+f^*(\bar{z}))^2} . \tag{6}$$

As follows from Eq. (6), L_{kin} can be rewritten so that only the field $T=f(z)$ appears in the kinetic term. Actually this holds for the whole Lagrangian. The considered theory depends only on the field T and all theories obtained by the replacement $T=f(z)$ are equivalent.

One expects that such a theory with a completely flat potential possesses an enlarged symmetry. For the case $T=(z+1)/(z-1)$ the scalar kinetic term becomes

$$L_{kin} = \frac{3|\partial_\mu z|^2}{(|z|^2-1)^2}$$

which is evidently invariant under the following set of transformations:

$$z \longrightarrow \frac{az+b}{b^*z+a^*} . \tag{7}$$

The set of transformations (7) forms the group $SU(1,1)$, which is non–compact and characterized by the parameters a and b which obey $|a|^2-|b|^2=1$. Hence $SU(1,1)$ is a three–dimensional group. Transformations of $SU(1,1)$ are defined by 2×2 matrices

$$U = \begin{pmatrix} a & b \\ b^* & a^* \end{pmatrix} ,$$

which can also be written in the form [7]

$$U = \exp\left\{ i\frac{\omega_0}{2}\sigma_3 + i\frac{\omega^*}{2}\sigma_- - i\frac{\omega}{2}\sigma_+ \right\}, \qquad \sigma_\pm = (\sigma_1 \pm i\sigma_2)/2 \tag{8}$$

Here ω_0 is a real parameter and $\sigma_{1,2,3}$ are the conventional Pauli matrices. The matrices U acting on the space $\begin{pmatrix} x \\ y \end{pmatrix}$ leave the element $|x|^2 - |y|^2$ invariant, in contrast with the $SU(2)$ group where we have invariance of the element $|x|^2 + |y|^2$. The $SU(1,1)$ transformations of the field variable T are

$$T \to \frac{(\alpha T + i\beta)}{(i\gamma T + \delta)} \qquad \alpha\delta + \beta\gamma = 1\,,$$

where α, β, γ and δ are real parameters.

The group $SU(1,1)$ contains the following subgroups [8]:

$$\begin{array}{lll} i) & \text{Imaginary translations:} & T \to T + i\beta; \\ ii) & \text{Dilatations:} & T \to \alpha^2 T; \\ iii) & \text{Conformal transformations:} & T \to \dfrac{\cos\theta T + i\sin\theta}{i\sin\theta T + \cos\theta}\,. \end{array} \tag{9}$$

The Kähler function (5) is invariant under the first set of transformations, but not under dilatations and conformal transformations. The gravitino mass term in the SUGRA Lagrangian, which appears when SUSY is broken, results in the breaking of $SU(1,1) \to U_a(1)$, where $U_a(1)$ is a subgroup of imaginary translations. One can wonder whether $SU(1,1)$ invariance implies a flat potential. The invariance of the scalar potential with respect to imaginary translations implies that $V(z,\bar{z})$ is a function of the sum $z + \bar{z}$. At the same time the invariance under dilatation forces $V(z,\bar{z})$ to depend only on the ratio $z/\bar{z}$. These two conditions are incompatible unless $V(z,\bar{z})$ is a constant. Moreover the $SU(1,1)$ invariance requires this constant to be zero [8]. In order to get a flat non–zero potential in SUGRA models, one should break $SU(1,1)$. The $SU(1,1)$ structure of the $N=1$ SUGRA Lagrangian can have its roots in supergravity theories with extended supersymmetry ($N=4$ or $N=8$) [3].

Let us consider a SUGRA model in which there are n chiral multiplets z and φ_i, $i = 1, 2, ...n-1$, where z is a singlet field while φ_i are non–singlets under the gauge group. If the Kähler function has the form

$$G = -3\ln\left(f(z) + f^*(\bar{z}) + g(\varphi_i, \bar{\varphi}_i)\right)\,, \tag{10}$$

then the F–part of the scalar potential vanishes and only D–terms give a non–zero contribution, so that

$$V = \frac{1}{2}\sum_a (D^a)^2\,, \qquad D^a = g_a \sum_{i,j}\left(G_i T^a_{ij}\varphi_j\right)\,, \tag{11}$$

where g^a is the gauge coupling constant associated with the generator T^a of the gauge transformations. Owing to the particular form of the Kähler function (10), the scalar potential (11) is positive definite. Its minimum is attained at the points for which $< D^a >= 0$ and the vacuum energy density vanishes [9].

In the case when $g(\varphi_i, \bar{\varphi}_i) = -\sum_i \varphi_i \bar{\varphi}_i$, the kinetic terms of the scalar fields are invariant under the isometric transformations of the non–compact $SU(n,1)$ group [9]. The manifestation of the extended global symmetry of L_{kin} can be clearly seen, if one uses new field variables y_i, i=0,1,...n-1, related to $f(z)$ and φ_i by

$$f(z) = \frac{1-y_0}{2(1+y_0)}\,, \qquad \varphi_i = \frac{y_i}{1+y_0}\,.$$

Then the Kähler function takes the form

$$G = -3\ln\left(1 - \sum_{i=0}^{n-1} y_i \bar{y}_i\right) + 3\ln|1+y_0|^2\,,$$

from which it follows that the kinetic terms of the scalar fields are

$$L_{kin} = \sum_j \frac{3\partial_\mu y_j \partial_\mu \bar{y}_j}{\left(1 - \sum_i y_i \bar{y}_i\right)^2}\,. \tag{12}$$

In particular the kinetic terms (12) remain intact if

$$y_i \to \frac{a_i y_i + b_i}{b_i^* y_i + a_i^*}\,; \qquad y_j \to \frac{y_j}{b_i^* y_i + a_i^*} \quad \text{for} \quad i \neq j\,, \tag{13}$$

where $|a_i|^2 - |b_i|^2 = 1$. The $SU(n,1)$ symmetry implies a zero contribution of the F–terms to the potential, which protects the vacuum energy density.

The $SU(n,1)$ symmetry can be derived from an extended ($N \geq 5$) supergravity theory [10]. This symmetry is broken by the gauge interactions (D–terms) in $N=1$ supergravity models, leaving only an $SU(1,1)$ symmetry. In terms of the symmetry transformations (13), the kinetic terms and scalar potential are still invariant with respect to the replacement

$$y_0 \to \frac{a_0 y_0 + b_0}{b_0^* y_0 + a_0^*}\,; \qquad y_i \to \frac{y_i}{b_0^* y_0 + a_0^*} \quad \text{for} \quad i \neq 0\,. \tag{14}$$

The gravitino mass breaks $SU(1,1)$ further to $U_a(1)$, since the Kähler function (10) is not invariant under the dilatation subgroup.

4. No–scale models with nontrivial superpotential and MPP

The introduction of the superpotential complicates the analysis. Suppose that the Kähler potential K of the model is given by Eq. (10) and the superpotential does not depend on the singlet superfield z. Then one can define the vector α_i

$$\alpha_i = e^{-K/3}\left[\frac{1}{3}F_i(\varphi_\alpha) - \frac{3+\sum_j g_{\bar{j}}(\varphi_\alpha,\bar{\varphi}_\alpha)F_j(\varphi_\alpha)}{3|\partial_z f(z)|^2}f_i(z)\right], \qquad (15)$$

where $F(\varphi_\alpha) = \ln W(\varphi_\alpha)$ and the indices i and j on the functions $f(z)$, $g(\varphi_\alpha,\bar{\varphi}_\alpha)$ and $F(\varphi_\alpha)$ denote the derivatives with respect to z and φ_α. The vector α_i satisfies the following property

$$\sum_j G_{i\bar{j}}\alpha_j = G_i$$

from which one deduces that

$$\sum_{i,k} G_i G^{i\bar{k}} G_{\bar{k}} = \sum_k \alpha_k G_{\bar{k}}\,. \qquad (16)$$

As a result the scalar potential takes the form

$$V = \frac{1}{3}e^{2K/3}\sum_\alpha \left|\frac{\partial W(\varphi_\alpha)}{\partial\varphi_\alpha}\right|^2 + \frac{1}{2}\sum_a (D^a)^2\,. \qquad (17)$$

The potential (17) leads to a supersymmetric particle spectrum at low energies. It is positive definite and its minimum is reached when $\left\langle\frac{\partial W(\varphi_\alpha)}{\partial\varphi_\alpha}\right\rangle = \, < D^a >= 0$, so that the cosmological constant goes to zero.

It is interesting to investigate what kind of symmetries protect the cosmological constant when $W(z,\varphi_\alpha) \neq const$. As discussed above, it is natural to seek such symmetries within the subgroups of $SU(1,1)$. The invariance of the Kähler function under the imaginary translations of the hidden sector superfields

$$z_i \to z_i + i\beta_i\,; \qquad \varphi_\alpha \to \varphi_\alpha \qquad (18)$$

implies that the Kähler potential depends only on $z_i + \bar{z}_i$, while the superpotential is given by

$$W(z_i,\varphi_\alpha) = \exp\left\{\sum_{i=1}^m a_i z_i\right\}\tilde{W}(\varphi_\alpha)\,, \qquad (19)$$

where a_i are real. Here we assume that the hidden sector involves m singlet superfields. Since $G(\phi_M,\bar{\phi}_M)$ does not change if

$$\begin{cases} K(\phi_M,\bar{\phi}_M) \to K(\phi_M,\bar{\phi}_M) - g(\phi_M) - g^*(\bar{\phi}_M)\,, \\ W(\phi_M) \to e^{g(\phi_M)} W(\phi_M) \end{cases} .$$

the most general Kähler function can be written as

$$G(\phi_M,\bar{\phi}_M) = K(z_i+\bar{z}_i,\varphi_\alpha,\bar{\varphi}_\alpha) + \ln|W(\varphi_\alpha)|\,, \tag{20}$$

where $W(\varphi_\alpha)=\tilde{W}(\varphi_\alpha)$.

The dilatation invariance constrains the Kähler potential and superpotential further. Suppose that hidden and observable superfields transform differently

$$z_i \to \alpha^k z_i\,, \qquad \varphi_\sigma \to \alpha\varphi_\sigma\,. \tag{21}$$

Then the superpotential $W(\varphi_\alpha)$ may contain either bilinear or trilinear terms involving the chiral superfields φ_α but not both. Because in phenomenologically acceptable theories the masses of the observable fermions are generated by trilinear terms, all others should be omitted. If there is only one field T in the hidden sector, then the Kähler function is fixed uniquely by the gauge invariance and symmetry transformations (18) and (21):

$$\begin{gathered} K(T+\bar{T},\varphi_\sigma,\bar{\varphi}_\sigma) = -\frac{6}{k}\ln(T+\bar{T}) + \sum_\sigma C_\sigma \frac{|\varphi_\sigma|^2}{(T+\bar{T})^{2/k}} \\ W(\varphi_\alpha) = \sum_{\sigma,\beta,\gamma} \frac{1}{6} Y_{\sigma\beta\gamma}\varphi_\sigma\varphi_\beta\varphi_\gamma\,, \end{gathered} \tag{22}$$

where C_σ and $Y_{\sigma\beta\gamma}$ are constants. The scalar potential of the hidden sector induced by the Kähler function, with K and W given by Eq. (22), is

$$V(T+\bar{T}) = \frac{3}{(T+\bar{T})^{6/k}}\left[\frac{2}{k}-1\right]$$

and vanishes when $k=2$. In this case the subgroups of $SU(1,1)$ — imaginary translations and dilatations ($T\to\alpha^2 T$, $\varphi_\sigma\to\alpha\varphi_\sigma$) — keep the value of the cosmological constant equal to zero.

The invariance of the Kähler function with respect to imaginary translations and dilatations prevents the breaking of supersymmetry. In order to demonstrate this, let us consider the SU(5) SUSY model with one field in the adjoint representation Φ and with one singlet field S. The superpotential that preserves gauge and global symmetries has the form

$$W(S,\Phi) = \frac{\varkappa}{3}S^3 + \lambda\mathrm{Tr}\Phi^3 + \sigma S\mathrm{Tr}\Phi^2\,. \tag{23}$$

In the general case the minimum of the scalar potential, which is induced by the superpotential (23), is attained when $< S >=< \Phi >= 0$ and does not lead to the breaking of local supersymmetry or of gauge symmetry. But if $\varkappa = -40\sigma^3/(3\lambda^2)$ there is a vacuum configuration

$$< \Phi >= \frac{\Phi_0}{\sqrt{15}} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -3/2 & 0 \\ 0 & 0 & 0 & 0 & -3/2 \end{pmatrix}, \qquad \begin{array}{c} < S >= S_0 \,, \\ \\ \Phi_0 = \dfrac{4\sqrt{15}\sigma}{3\lambda} S_0 \,, \end{array} \tag{24}$$

which breaks SU(5) down to $SU(3) \times SU(2) \times U(1)$. However, along the valley (24), the superpotential and all auxiliary fields F_i vanish preserving supersymmetry and the zero value of the vacuum energy density.

In order to get a vacuum where local supersymmetry is broken, one should violate dilatation invariance, allowing the appearance of the bilinear terms in the superpotential of SUGRA models. Eliminating the singlet field from the considered SU(5) model and introducing a mass term for the adjoint representation, we get

$$W(\Phi) = M_X \mathrm{Tr}\Phi^2 + \lambda \mathrm{Tr}\Phi^3 \,. \tag{25}$$

In the resulting model, there are a few degenerate vacua with vanishing vacuum energy density. For example, in the scalar potential there exists a minimum where $< \Phi >= 0$ and another vacuum, which has a configuration similar to Eq. (24) but with $\Phi_0 = \dfrac{4\sqrt{15}}{3\lambda} M_X$. In the first vacuum the SU(5) symmetry and local supersymmetry remain intact, while in the second one the auxiliary field F_T acquires a vacuum expectation value and a non-zero gravitino mass is generated:

$$\begin{aligned} < |F_T| > \quad &\simeq \quad \left\langle \frac{|W(\Phi)|}{(T+\bar{T})^{1/2}} \right\rangle \quad = \quad m_{3/2}(T+\bar{T}) \,, \\ m_{3/2} \quad &= \quad \left\langle \frac{|W(\Phi)|}{(T+\bar{T})^{3/2}} \right\rangle \quad = \quad \frac{40}{9} \frac{M_X^3}{\lambda^2 (T+\bar{T})^{3/2}} \,. \end{aligned} \tag{26}$$

As a result, local supersymmetry and gauge symmetry are broken in the second vacuum. However it does not break global supersymmetry in the observable sector at low energies (see Eq.(17)). When M_X goes to zero the dilatation invariance, SU(5) symmetry and local supersymmetry are restored.

A simple model with the superpotential (25) can serve as a basis for the Multiple Point Principle (MPP) assumption in SUGRA models, which was formulated recently in [11]. When applied to supergravity, MPP implies

that the scalar potential contains at least two degenerate minima. In one of them local supersymmetry is broken in the hidden sector, inducing a set of soft SUSY breaking terms for the observable fields. In the other vacuum the low energy limit of the considered theory is described by a pure supersymmetric model in flat Minkowski space. Since the vacuum energy density of supersymmetric states in flat Minkowski space is just zero, the cosmological constant problem is thereby solved to first approximation by the MPP assumption. An important point is that the vacua with broken and unbroken local supersymmetry are degenerate and have zero energy density in the model considered above. However, in the vacuum where local supersymmetry is broken, all soft SUSY breaking terms vanish making this model irrelevant for phenomenological studies.

5. No–scale models and the superstring

The Kähler function and the structure of the hidden sector should be fixed by an underlying renormalizable or even finite theory. Nowadays the best candidate for the ultimate theory is $E_8 \times E_8$ (ten dimensional) heterotic superstring theory [12]. The minimal possible SUSY–breaking sector in string models involves dilaton (S) and moduli (T_m) superfields. The number of moduli varies from one string model to another. But dilaton and moduli fields are always present in four–dimensional heterotic superstrings, because S is related with the gravitational sector while vacuum expectation values of T_m determine the size and shape of the compactified space. Amongst the moduli T_m we concentrate here on the overall modulus T. In this case Calabi–Yau and orbifold compactifications lead to rather similar results for the Kähler potential, superpotential and gauge kinetic functions at the tree level:

$$\begin{gathered} K = -\ln(S+\bar{S}) - 3\ln(T+\bar{T}) + \textstyle\sum_\alpha (T+\bar{T})^{n_\alpha} \varphi_\alpha \bar{\varphi}_\alpha\,, \\ W = W^{(ind)}(S,\,T,\,\varphi_\alpha) + \sum_{\sigma,\beta,\gamma} \frac{1}{6} Y_{\sigma\beta\gamma} \varphi_\sigma \varphi_\beta \varphi_\gamma\,, \qquad f_a = k_a S\,, \end{gathered} \tag{27}$$

where k_a is the Kac–Moody level of the gauge factor ($k_3 = k_2 = \frac{3}{5} k_1 = 1$). In the case of orbifold compactifications, the n_α are negative integers sometimes called modular weights of the matter fields. Orbifold models have a symmetry ("target–space duality") which is either the modular group $SL(2,\mathbf{Z})$ or a subgroup of it. Under $SL(2,\mathbf{Z})$, the fields transform like

$$\begin{gathered} T \to \frac{aT - ib}{icT + d}\,, \qquad ad - bc = 1 \quad a,\,b,\,c,\,d\,\in\,\mathbf{Z}; \\ S \to S\,; \qquad \varphi_\alpha \to (icT+d)^{n_\alpha} \varphi_\alpha\,. \end{gathered} \tag{28}$$

In the large T limit of the Calabi–Yau compactifications, $n_\alpha = -1$ and the Lagrangian of the effective SUGRA models is also invariant with respect to the field transformations (28) if $n_\alpha = -1$. So one can see that the form of the Kähler function is very close to the no–scale structure discussed in the previous sections.

In the classical limit $W^{(ind)}(S, T, \varphi_\alpha)$ is absent. The superpotential of the hidden sector and supersymmetric mass terms of the observable superfields may be induced by non-perturbative corrections, which violate the invariance under $SL(2, \mathbf{Z})$ symmetry. In the gaugino condensation scenario for SUSY breaking, the superpotential of the hidden sector takes the form:

$$W(S, T) \sim \exp\{-3S/2b_Q\} \,, \tag{29}$$

where b_Q is the beta–function of the hidden sector gauge group. For an $SU(N)$ model without matter superfields $b_Q = 3N/(16\pi^2)$. Assuming that the superpotential does not depend on T, we get

$$V(S, T) = \frac{1}{(S+\bar{S})(T+\bar{T})^3}\left|\frac{\partial W(S)}{\partial S} - \frac{W(S)}{S+\bar{S}}\right|^2 . \tag{30}$$

The scalar potential (30) of the hidden sector is positive definite. All its vacua are degenerate and have zero energy density. Among them there can be a minimum where the vacuum expectation value of the hidden sector superpotential vanishes. It is easy to check that, in this vacuum, local supersymmetry remains intact. In other vacua where $< W(S) > \neq 0$ local supersymmetry is broken, since $F_T \neq 0$. Thus the MPP conditions can be realized in superstring inspired models as well.

But at low energies the SUGRA Lagrangian, corresponding to the Kähler function given by Eq. (27) with $n_\alpha = -1$ and a superpotential that does not depend on the overall modulus T, exhibits structure inherent in global supersymmetry. In order to destroy the degeneracy between bosons and fermions, the $SL(2, \mathbf{Z})$ symmetry should be broken further. Non-zero gaugino masses M_a are generated when the gauge kinetic function gets a dependence on T, i.e. $f_a = k_a(S - \sigma T)$. The soft scalar masses m_α^2 and trilinear couplings $A_{\alpha\beta\gamma}$ arise for the minimal choice of the Kähler metric of the observable superfields, when the Kähler potential is given by

$$K = -\ln(S+\bar{S}) - 3\ln(T+\bar{T}) + \sum_\alpha \varphi_\alpha \bar{\varphi}_\alpha \,. \tag{31}$$

In this case we have

$$A_{\alpha\beta\gamma} = 3m_{3/2}\,, \qquad m_\alpha^2 = m_{3/2}^2, \tag{32}$$

It is worth emphasizing that the energy densities of vacua still vanish in models with the modified gauge kinetic function and Kähler potential (31). It clears the way to the construction of realistic SUGRA models based on the MPP assumption.

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References

[1] A.G.Riess *et al.*, Astron.J. **116**, 1009 (1998); S.Perlmutter *et al.*, Astrophys.J. **517**, 565 (1999); C.Bennett *et al.*, astro-ph/0302207; D.Spergel *et al.*, astro-ph/0302209.

[2] J.Polonyi, Budapest preprint KFKI–1977 93 (1977).

[3] A.B.Lahanas, D.V.Nanopoulos, Phys.Rep. 145 (1987) 1.

[4] D.L.Bennett, H.B.Nielsen, Int.J.Mod.Phys. A **9**, 5155 (1994); *ibid* **14**, 3313 (1999); D.L.Bennett, C.D.Froggatt, H.B.Nielsen, in *Proceedings of the 27th International Conference on High energy Physics, Glasgow, Scotland, 1994*, p.557; *Perspectives in Particle Physics '94, World Scientific, 1995*, p. 255, ed. D. Klabučar, I. Picek and D. Tadić [arXiv:hep-ph/9504294].

[5] H.P.Nilles, Phys.Rep. **110**, 1 (1984).

[6] E.Gremmer, S.Ferrara, C.Kounnas, D.V.Nanopoulos, Phys.Lett. B 133 (1983) 61.

[7] J.Ellis, M.K.Gaillard, M.Günaydin, B.Zumino, Nucl.Phys. B 224 (1983) 427.

[8] J.Ellis, C.Kounnas, D.V.Nanopoulos, Nucl.Phys. B 241 (1984) 406.

[9] J.Ellis, C.Kounnas, D.V.Nanopoulos, Nucl.Phys. B 247 (1984) 373.

[10] E.Gremmer, B.Julia, Phys.Lett.B 80 (1978) 48; Nucl.Phys.B 159 (1979) 141.

[11] C.Froggatt, L.Laperashvili, R.Nevzorov, H.B.Nielsen, Phys.Atom.Nucl.67 (2004) 582.

[12] M.B.Green, J.H.Schwarz, E.Witten, *Superstring Theory, Cambridge Univ. Press, Cambridge, 1987.*